Effects of mass renormalization on the surface properties of heavy-ion fusion potential

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Abstract

We discuss the effects of fast nuclear excitations on heavy-ion fusion reactions at energies near and below the Coulomb barrier. Using the fusion of two ^{40}Ca nuclei as an example and the inversion method, we show that the mass renormalization induced by fast nuclear excitations leads to a large surface diffuseness in the effective potential for heavy-ion fusion reactions.

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Heavy-ion fusion reactions at energies below the Coulomb barrier are typical examples of the macroscopic quantum tunneling, which has been a very popular subject in the past decade in many fields of physics and chemistry[1, 2, 3]. One of the major interests in the macroscopic quantum tunneling is to assess the effects of environments on the tunneling rate of a macroscopic variable. When the bombarding energy is below the Coulomb barrier, heavy-ion fusion reactions take place by a quantum tunneling. It is now well established that the fusion cross section at sub-barrier energies is enhanced by several orders of magnitude compared with the prediction of a one-dimensional potential model due to the coupling of the relative motion to nuclear internal degrees of freedom, which play the role of environments[4]. The present authors have shown that the coupling of the tunneling degree of freedom to fast environmental degrees of freedom causes both a static potential and a mass renormalizations[5]. On the other hand, the first and the second derivatives of the experimental fusion cross section with respect to the bombarding energy suggests that the fusion reaction between two ^{40}Ca nuclei can be described by a potential model using the surface diffuseness parameter a=1.27 fm[6, 7], which is much larger than the usually accepted value $a \sim 0.6$ fm. The spin distribution of the compound nucleus made by the fusion of ^{16}O with ^{154}Sm also suggests a large effective surface diffuseness parameter a=1.27 fm[8]. Such a large surface diffuseness has already been suggested long time ago through the inversion method of obtaining the effective fusion potential directly from the experimental data. In that procedure, a much larger surface diffuseness a=1.59 fm was suggested for ^{40}Ca + ^{40}Ca scattering [9].

The aim of this paper is to discuss the connection between this large effective surface diffuseness and the potential and the mass renormalizations induced by the coupling of the relative motion between heavy ions with fast nuclear intrinsic degrees of freedom. We consider the fusion of two ^{40}Ca nuclei as an example. We first determine the bare nuclear

potential in the entrance channel by the double folding procedure. We then calculate the excitation function of the fusion cross section by taking the effects of a few vibrational modes of excitation of ^{40}Ca into account through the adiabatic potential as well as the mass renormalizations. We then use the inversion procedure to obtain the effective fusion potential from thus obtained excitation function of the fusion cross section and discuss its surface properties.

In the double folding procedure, one often expresses the densities of nuclei by a step function of range R_d convoluted with a Yukawa smearing function of range $1/\kappa_d$ and assume a Yukawa interaction for the nucleon-nucleon interaction with the range parameter κ_I. The resulting potential consists of two parts, which are proportional to $e^{-\kappa_d R}$ and $e^{-\kappa_I R}$ in the surface region[10]. If the Michigan three range Yukawa (M3Y) interaction[11, 12] is assumed as is often done for the nucleon-nucleon interaction, κ_I is larger than the standard value of κ_d. The diffuseness parameter of the double folding potential is then approximately given by the inverse of κ_d. More precisely, the surface diffuseness parameter slightly deviates from this simple consideration. Actually, Akyüz and Winther numerically performed the double folding procedure and obtained the value 1/1.36 fm for the surface diffuseness parameter in the potential between two ^{40}Ca nuclei when they assumed the charge distribution of ^{40}Ca to be given by κ_d=1.55 fm^{-1} and used the M3Y force as the nucleon-nucleon interaction[13]. Note that 1/1.36=0.75 fm is slightly larger than the standards of what is usually used in optical-model calculations, but not so much as those suggested from the analyses of the experimental sub-barrier fusion cross section. Henceforth we take the Woods-Saxon shape for the bare nuclear potential between two ^{40}Ca nuclei

$$V_N(R) = -\frac{V_0}{1 + e^{(R-2R_0)/a}} \tag{1}$$

and use the value 0.75 fm for the surface diffuseness parameter a as suggested in the above

arguments. Following ref.[10], the strength and the range of the bare nuclear potential are taken to be

$$V_0 = 16\pi\gamma a\bar{R} \tag{2}$$

$$R_0 = 1.2 \cdot 40^{1/3} - 0.09, \tag{3}$$

where γ and $\bar{R}$ are 0.95 and $R_0/2$, respectively. The bare nuclear potential thus obtained gives the Colulomb barrier at 10.0fm with the height and the curviture 53.1 MeV and 3.43 MeV, respectively.

We now consider the effects of surface vibrations of ^{40}Ca during the fusion process. We ignore the effects of nucleon transfer reactions, althogh they play an important role in quantitatively explaining the experimental fusion cross section [14, 15, 16]. Following ref.[14] we consider the excited states of ^{40}Ca at 3.74, 3.91, and 4.49 MeV with spin-parity 3^-, 2^+ and 5^- to be the one phonon state of three different vibrational modes of excitation, though the 2^+ state will rather be the 2^+ member of the rotational band based on the 0^+ state at 3.35 MeV[17]. Our interest is to study the effects of the coupling of the relative motion to these states on the surface diffuseness parameter in the effective potential for fusion reactions.

We assume the linear oscillator coupling and use the no-Colioris approximation in order to reduce the dimension of the coupled-channels equations[18, 19]. The effective Hamiltonian for this system then reads

$$
\begin{aligned}
H = & -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial R^2} + \frac{2}{R}\frac{\partial}{\partial R}\right) + \frac{J(J+1)\hbar^2}{2MR^2} + V_C(R) + V_N(R) \\
& + \sum_{\lambda=3,2,5}\left\{\hbar\omega_\lambda\left(a_\lambda^\dagger a_\lambda + \frac{1}{2}\right) + \sqrt{\frac{2\lambda+1}{4\pi}}\left(\alpha_{\lambda 0}^{(N)}f_N(R) + \alpha_{\lambda 0}^{(C)}f_C(R)\right)\left(a_\lambda^\dagger + a_\lambda\right)\right\}
\end{aligned} \tag{4}
$$

where M, R, J, V_C, V_N, and λ are the reduced mass, the relative distance, the total angular momentum, the Coulomb and the nuclear interactions between the two ^{40}Ca nuclei, and

the multipolalities of the vibrational excitations, respectively. $f_N(R)$ and $f_C(R)$ are the nuclear and the Coulomb coupling form factors, respectively. We determine them following the collective model[10]. $\hbar\omega_\lambda, a_\lambda^\dagger(a_\lambda), \alpha_{\lambda 0}^{(N)}$, and $\alpha_{\lambda 0}^{(C)}$ are the excitation energy, the creation (annihilation) operators of the oscillator quanta, the nuclear and the Coulomb coupling strenghes of the 2^λ-pole vibrational excitations, respectively. The factor $\sqrt{\frac{2\lambda+1}{4\pi}}$ in front of the coupling strength arises from the no-Coriolis approximation. We take the coupling strenghes from ref.[14] (see Table I). In Table I, R_C and R_N are the charge and the matter radii of ^{40}Ca, respectively. The Coulomb coupling strength was determined from the transition strength $B(E\lambda)$ from the one phonon state of each vibrational mode of excitation to the ground state. The nuclear coupling strength was modified to reproduce the data of inelastic scattering of ^{16}O from ^{40}Ca. All the coupling strenghes are multipled by $\sqrt{2}$ in order to take the mutual excitations of the target and the projectile into account.

The excitation energy $\hbar\omega$ of all the vibrational excitations which we are interested in now is larger than the curvature of the bare potential barrier. Therefore, the present problem corresponds to the case of an adiabatic tunnelig, i.e. a slow tunneling. The adiabaticity of the tunneling process depends on two parameters[20]. One is the ratio of the energy scale $a_1 = \hbar\Omega/\hbar\omega$, $\hbar\Omega$ being the curvature of the bare potential barrier. The other is the ratio of the strength of the coupling Hamiltonian to the excitation energy $a_2 = \alpha_0 c R_a/\hbar\omega$, R_a and $\alpha_0 c$ being the thickness of the tunneling region and the derivative of the coupling Hamiltonian in the case when one assumes a bi-linear coupling, respectively. The adiabatic assumption breaks down if these parameters are larger than one. The typical values of these parameters for ^{40}Ca + ^{40}Ca scattering are listed in Table II. In order to obtain a_2, we estimated the derivative of the coupling Hamiltonian at the position of the bare potential barrier and determined R_a at energy 1 MeV below the top

of the bare potential barrier based on the parabolic approximation. Table II justifies the use of the adiabatic formula to calculate the barrier penetrability for the fusion reaction in the present system. We use the revised adiabatic formula of ref.[5]. It modifies the well known adiabatic formula, which takes only the static potential shift into account, by adding the mass renormalization. Our formula for the barrier penetrability then reads

$$P(E) = \frac{1}{1 + e^{2S(E)}} \tag{5}$$

with

$$S(E) = \int_{R_1}^{R_2} dR \sqrt{\frac{2M_{ad}(R)}{\hbar^2} (V_{ad}(R) - E)} \tag{6}$$

for the s-wave scattering. In eq.(6) R_1 and R_2 are the inner and the outer classical turning points, respectively. The renormalized potential and mass in this equation are given by [5].

$$V_{ad}(R) = V_C(R) + V_N(R) - \sum_{\lambda=3,2,5} \frac{2\lambda + 1}{4\pi} \left(\alpha_{\lambda 0}^{(N)} f_N(R) + \alpha_{\lambda 0}^{(C)} f_C(R) \right)^2 \frac{1}{\hbar \omega_\lambda} \tag{7}$$

$$M_{ad}(R) = M + 2 \sum_{\lambda=3,2,5} \frac{2\lambda + 1}{4\pi} \left(\alpha_{\lambda 0}^{(N)} \frac{df_N(R)}{dR} + \alpha_{\lambda 0}^{(C)} \frac{df_C(R)}{dR} \right)^2 \frac{1}{\hbar \omega_\lambda^3} \tag{8}$$

Fig.1 shows the tunneling probability in the s-wave scattering as a function of the bombarding energy. The dotted line is the penetrability when there is no coupling, i.e. the penetrability for the bare potential barrier with the bare mass M. The dashed line is obtained with the standard adiabatic formula, where only the adiabatic potential renormalization is taken into account. The result of the revised adiabatic formula which takes the mass renormalization into account is denoted by the solid line. Notice that the overenhancement of the effects of the channel coupling when one considers only the adiabatic potential shift (the dashed line) is corrected by the effects of the mass renormalization. Fig.2 shows the renormalized mass $M_{ad}(R)$ in this system (eq.(8)) in the ratio to the bare mass M. It is a function of the separation distance between the colliding nuclei reflecting

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the radial dependence of the coupling form factor. It has two peaks because of the strong cancellation between the Coulomb and the nuclear couplings. The small spike at around R=8fm originates from the discontinuity of the derivative of the Coulomb coupling form factor. This defect plays, however, no essential role.

We now derive the effective fusion potential corresponding to the solid line in Fig.1. We thus demonstrate how the effects of the channel coupling including the mass renormalization can be mocked up by renormalizing the surface diffuseness. To this end, we use the inversion procedure which gives the thickness of the tunneling region, i.e. the thickness of the potential barrier, at each energy directly from the excitation function of the fusion cross section[9]

$$t(E) = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2M}}\int_E^B \frac{\frac{dS(E')}{dE'}}{\sqrt{E'-E}}dE' \tag{9}$$

where $S(E)$ is the action integral given by eq.(6) and B is the barrier height detemined from the condition $S(B) = 0$. This is the height of the potential barrier including the adiabatic potential shift. Note that the mass M in the factor in front of the integral in eq.(9) does not include the mass renormalization, but the bare mass. This is because our aim is to get the effective potential for the relative motion with bare mass. The results of the inversion procedure are shown in fig.3 by the solid line. For the integration in eq.(9), we changed the variable from E' to $\sqrt{E'-E}$ to eliminate the singularity at the lower limit of the integral[9]. The inversion procedure gives only the barrier thickness $t(E)$. Consequently, the turning points themselves as functions of the energy remain indeterminate. We assumed that the outer turning points are given by those of the adiabatic potential barrier and determine the inner turning points of the effective potential barrier by using the barrier thickness given by the inversion procedure. For comparison, fig.3 also shows the bare potential barrier (the dotted line) and the adiabatic potential barrier (the dashed line).

Since we are interested in the surface diffuseness parameter of the nuclear potential, we subtract the Coulomb potential $V_C(R)$ from both the effective potential obtained by the inversion procedure and the adiabatic potential. We call these potentials the effective nuclear potentials, because they contain not only the effects of the nuclear coupling but also those of the Coulomb coupling. Fig.4 shows the effective nuclear potentials obtained in this way. The dashed line denotes the effective nuclear potential in the adiabatic limit, i.e., the potential obtained by subtracting the Coulomb potential from the adiabatic potential. The effective nuclear potential, which takes account of the mass renormaliztion as well, is denoted by the solid line. Also shown are the potentials of an exponential shape with two different surface diffuseness parameters. Their strength was chosen, such that the values of the resultant potentials agree with that of the adiabatic potential at the peak position. The dot-dashed and the dotted lines correspond to the potential when the surface diffuseness parameter was chosen to be 0.7 and 0.8 fm, respectively. These choices simulate the effective potential in the adiabatic limit (the dashed line), and that which includes the effects of the mass renormalization(solid line). Remember that the surface diffuseness parameter for the bare potential is about 0.75 fm. Fig.4 shows that the surface diffuseness parameter is smaller than that of the bare nuclear potential if only the effects of the adiabatic potential shift is taken into account, but gets larger than that of the bare nuclear potential when the effects of the mass renormalization is added. We thus conclude that one of the origins of the large surface diffuseness of the effective nuclear potential in heavy-ion fusion reactions is the effects of the coupling to nuclear inelastic excitations, which lead to both the potential and the mass renormalizations.

In summary, we discussed the effects of nuclear intrinsic excitations on heavy-ion fusion reactions. We recast the potential and the mass renormalizations due to fast inelastic excitations of scattering nuclei in terms of the renormalized surface diffuseness in the

effective fusion potential. As a concrete example, we analyzed the fusion reactions between two ^{40}Ca nuclei. We used the inversion method to obtain the effective nuclear potential and found that the surface diffuseness parameter becomes larger than that obtained by the double folding method if both the adiabatic potential shift and the adiabatic mass renormalization are taken into account. The value of the surface diffuseness parameter we obtained is about 0.8 fm compared to the standard value 0.6 fm. Although this value is not sufficiently large to explain the large surface diffuseness empirically obtained from the experimental data of sub-barrier fusion cross sections, we conclude that the large surface diffuseness obtained from data analyses partly represents the effects of channel coupling. It would be interesting to study how the effects of transfer reactions further modify the surface properties of the effective fusion potential. The large surface diffuseness parameter has also been suggested in the fusion reactions of ^{16}O on ^{154}Sm, which is a fast tunneling rather than a slow tunneling discussed in this paper[8]. It remains an open question how we should take into account the effects of the channel coupling in such a system to explain the large surface diffuseness. Contrary to the case of slow tunneling discussed in this paper, one has to treat a distributed potential barriers, or an energy dependent effective potential barrier[21].

Table I

λ^π	$\hbar\omega_\lambda$(MeV)	$R_C\alpha_{\lambda0}^{(C)}$(fm)	$R_N\alpha_{\lambda0}^{(N)}$(fm)
3^-	3.737	0.6576	0.4455
2^+	3.905	0.1952	0.1768
5^-	4.492	0.4837	0.2475

Table II

λ^π	a_1	a_2
3^-	0.9177	0.6176
2^+	0.8783	0.1997
5^-	0.7635	0.3668

Table Captions

Table I: Low lying vibrational states of ^{40}Ca. $\hbar\omega_\lambda$, $\alpha_{\lambda0}^{(C)}$ and $\alpha_{\lambda0}^{(N)}$ are the excitation energy of the 2^λ-pole vibrational state, and its Coulomb and nuclear coupling strengthes, respectively. R_C and R_N are the charge and the matter radii, respectively.

Table II: Adiabaticity parameters for the tunneling process. a_1 is the ratio of the energy scales of the tunneling and the environmental degrees of freedom $\hbar\Omega/\hbar\omega$, while a_2 the ratio of the strengthes of the channel coupling Hamiltonian to the excitation energy of the environment $\alpha_0 cR_a/\hbar\omega$.

Figure Captions

Fig.1: Excitation function of the s-wave barrier penetrability for ^{40}Ca + ^{40}Ca scattering. The dotted line is the penetrability in the absence of the channel coupling. The dashed line was obtained by the standard adiabatic formula which takes only the effects of the adiabatic potential shift into account. The solid line was calculated with the revised adiabatic formula by adding the effects of the mass renormalization.

Fig.2: The renormalized mass (eq. (8)) as a function of the separation distance between the colliding nuclei in the unit of the bare mass.

Fig.3: Fusion barrier obtained by the inversion procedure. The dotted and the dashed lines are the bare and the adiabatic potential barriers, respectively. The solid line includes the effects of the mass renormalization.

Fig.4: Effective nuclear potential for the fusion of two ^{40}Ca nuclei. The solid line was obtained by subtracting the Coulomb potential $V_C(R)$ from the effective potential barrier in the presence of the mass renormalization. The dashed line is the effective nuclear potential in the adiabatic limit disregarding the mass renormalization. The dott-dashed and the dotted lines are the exponential potentials with the surface diffuseness parameters 0.7 and 0.8 fm, respectively. Their depth was chosen to reproduce the barrier height of the adiabatic potential. The surface diffuseness parameters a=0.7 fm and 0.8 fm were fixed to simulate the nuclear potential without (the dashed line) and with (the solid line) the mass renormalization.

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